STRICTION SELF-FOCUSING OF ELECTROMAGNETIC PULSES IN PLASMA

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As is well known [1], when electromagnetic waves propagate in plasma, self-focusing can arise as a result of the redistribution of plasma density under the action of the field. Such redistribution is possible both as a result of plasma heating as well as a result of electrostriction pressure. There are situations when the striction mechanism dominates. This occurs primarily for self-focusing in solids, where absorption of light is small. But, even in media with large absorption of light, the thermal pressure can exceed the striction pressure only for pulses with sufficiently long duration (exceeding 10^{-5}-10^{-6} sec [2-4]).

The system of equations describing the striction self-focusing, assuming that the perturbation of the density of the medium $\delta\rho$ is small, is examined in [5]. It is shown that a collapse, i.e., unbounded increase in the field amplitude E and density perturbation as the instant of collapse t_0 is approached, can occur in such a system and, in addition, the following relations are satisfied approximately:

$$E \sim (t_0 - t)^{-1},\ \delta\rho \sim (t_0 - t)^{-2}. \qquad (1)$$

Obviously, such an approach is valid as long as the nonlinearity remains small. It is clear that the decrease in density $\delta\rho$ cannot exceed the unperturbed density ρ_0.

The present work examines the striction self-focusing without assuming the smallness of $\delta\rho$ and the flow speed of the medium.

The system of equations that describe the striction self-focusing has the form [1]

$$\begin{gathered}\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r\rho v = 0, \quad \frac{\partial}{\partial t}\rho v + \frac{1}{r}\frac{\partial}{\partial r} r\rho v^2 = -\nabla p + F,\\ 2ik\left(\frac{\partial E}{\partial z} + \frac{1}{v_{\text{gr}}}\frac{\partial E}{\partial t}\right) + \Delta_\perp E + k^2\frac{\delta\varepsilon}{\varepsilon_0}E = 0,\\ F = \frac{1}{16\pi}\left\{\nabla\left(|E|^2\rho\frac{\partial\varepsilon}{\partial\rho}\right) - |E|^2\nabla\varepsilon\right\}, \qquad p = p(\rho),\end{gathered} \qquad (2)$$

where ρ, ν, ε, and p are the density, velocity, dielectric permeability, and pressure of the medium, respectively.

For an isotropic plasma

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}, \quad F = -\frac{e^2 n}{4m\omega^2}\nabla|E|^2, \qquad \rho = nM.$$

It is assumed here that the electron distribution is in equilibrium with the potential created by the striction pressure of the field, while the quantities ν and ρ correspond to the ionic component of the plasma.

The first two equations in this system do not take into account motion along the axis of the pulse, which is small in comparison with the radial motion.

In what follows, we will examine striction self-focusing in an isotropic plasma, assuming that $T_e \gg T_i$, and $P = nT_e$.

We will seek a solution to the system near the beam axis in the form [6]

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$$E = A(t')e^{i\nu}f(z',\ t')\exp(-r^2f^2(z',\ t')/a^2),$$
$$\rho = \rho_0 - g^2(z',\ t')N(z')\exp(-r^2g^2(z',\ t')/b^2), \tag{3}$$
$$v = v_0\varphi(z',\ t')r,$$

where $z' = z$; $t' = t - z/v_{gr}$; $A(t')$ determines the shape of the pulse; and g^2N is the density perturbation.

Since only z' and t' will be used below, in order to simplify the notation, we will omit the primes for these variables.

Substituting (3) into (2), we obtain for $f^2 \gg 1$ and $g^2 \gg 1$ the following equations for the functions f and g:

$$k^2f\left(\frac{1}{f}\right)_{zz} = \frac{4f^4}{a^4} + \frac{4\pi e^2k^2}{mM\omega^2}\frac{g^4N}{b^2},$$

$$\frac{3g'^2g^2N^2}{(\rho_0 - g^2N)^2} = \frac{T_e}{M}\frac{g^4N}{(\rho_0 - g^2N)\,b^2} + \frac{e^2A^2f^4}{2mM\omega^2a^2}.$$

Let us examine the case of slow variation of f along z, i.e. when it is possible to neglect the term $k^2f(1/f)_{zz}$. Then we have the equation

$$g'^2 = \frac{g^2}{3b^2N}\left(n_0T_e - \left(\frac{\omega_p}{\omega}\right)^4\frac{k^2A^2a^2}{4\cdot 8\pi}\left(1 - \frac{g^2N}{\rho_0}\right)\right)\left(1 - \frac{g^2N}{\rho_0}\right),$$
$$f^4 = -\frac{\pi e^2a^4}{mM\omega^2}\frac{g^4N}{b^2},\quad n_0 = \rho_0/M.$$

Let us consider rectangular pulses A = const. The form of the general solution to this equation depends on the sign of the quantity

$$c = n_0T_e - \left(\frac{\omega_p}{\omega}\right)^4\frac{k^2A^2a^2}{32\pi},$$

i.e. on the sign of the difference between the thermal pressure and the effective pressure of the electromagnetic field. For $c < 0$,

$$g^2 = 2c/\left(d - (d^2 - 4lc)^{1/2}\sin\left(\sqrt{-\alpha c}\,t + c_0\right)\right),$$
$$d = \left(\frac{\omega_p}{\omega}\right)^4\frac{k^2A^2a^2}{32\pi},\quad l = -\left(\frac{\omega_p}{\omega}\right)^4\frac{k^2A^2a^2}{32\pi},\quad \alpha = \frac{4}{3b^2N},$$

where c_0 is a constant of integration, i.e. when the striction pressure exceeds the thermal pressure, oscillation in the amplitude can arise. For $c > 0$,

$$g^2 = -\frac{4c\exp\left(-\sqrt{\alpha c}\,t + c_0\right)}{4lc - \{\exp\left(-\sqrt{\alpha c}t + c_0\right) - d\}^2}. \tag{4}$$

In the limit $\sqrt{\alpha c}\ t + c_0 < 1$, this expression reduces to the form

$$g^2 \sim 1/(1 - \sqrt{\alpha c}\,t).$$

In the opposite limiting case $\sqrt{\alpha c}\ t + c_0 > 1$

$$g^2 \sim 4c/\{(4lc - d^2)\exp(-\sqrt{\alpha c}\,t + c_0) - 2d\}.$$

Thus, during the initial stage, the field amplitude along the axis and the density perturbation will increase according to the laws

$$E \sim 1/(t_0 - t)^{1/2},\ \delta\rho \sim 1/(t_0 - t) \tag{5}$$

with a characteristic growth time t_0 equal in order of magnitude to

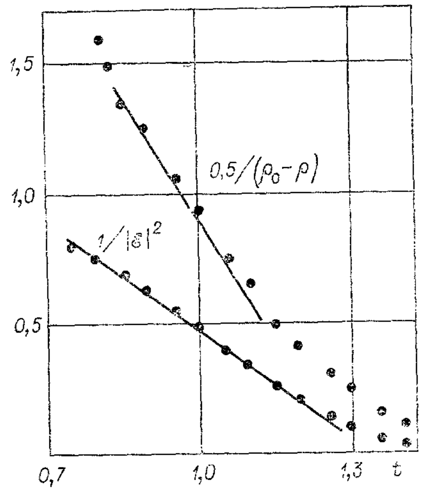

Fig. 1

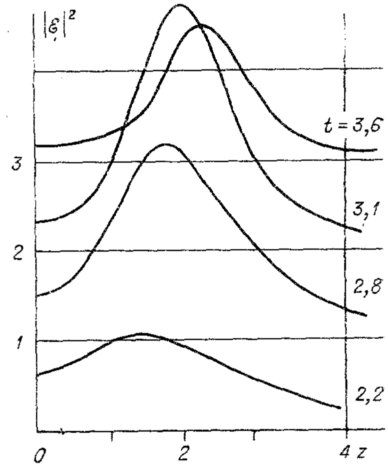

Fig. 2

$$t_0 \sim 1/\sqrt{\alpha c}.$$

As time passes, this increase must slow down and after attaining some maximum must become blurred. We note that for the strong nonlinearity examined here, the field increases in time more slowly than in the case of a weak nonlinearity (cf. (1) and (5)).

It is natural to expect that the results obtained can be reasonably applied when the quantities involved vary sufficiently slowly, i.e., far away from the instant of maximum collapse. But, qualitatively, the conclusions must be correct, a fact which is verified by the results of numerical simulation.

For numerical simulation, it is more convenient to transform to dimensionless variables:

$$\begin{aligned}
&\frac{\partial\rho}{\partial t}+\frac{\nu}{r}\frac{\partial}{\partial r}r\rho v=0,\\
&\frac{\partial}{\partial t}\rho v+\frac{\nu}{r}\frac{\partial}{\partial r}r\rho v^2=-\mu\frac{\partial\rho}{\partial r}-\varkappa\rho\frac{\partial}{\partial r}|\mathscr{E}|^2,\\
&2i\frac{\partial\mathscr{E}}{\partial z}+\Delta_\perp\mathscr{E}-\sigma\rho\mathscr{E}=0,
\end{aligned}\tag{6}$$

where $\nu=\frac{\tau u_0}{a}$; $\mu=\frac{T_e\tau}{Mu_0a}$; $\sigma=\frac{\omega_p^2}{\omega^2}k^2a^2$; $u_0=\frac{\omega_p^2}{\omega^2}\frac{E_0^2\tau}{(4\pi)^2Man_0}$; E_0, n_0, τ, and a are the characteristic field amplitude, plasma density, duration, andtransverse dimensions of the pulse, respectively.

In the numerical calculations performed, the variables in the system (6) are measured in terms of $E_0 = 2\cdot 10$ CGSE units, $n_0 = 5\cdot 10^{15}$ cm^{-3}, $\tau = 10^{-6}$ sec, $a = 0.5$ cm, and $\omega_p^2/\omega^2 = 1/2$.

Figure 1 shows the variation of the maximum values of the field and the density as a function of time. The behavior of $\mathscr{E}$ and ρ, as can be seen, verifies the ideasdeveloped above and the initial stage of self-focusing corresponds to the self-similar law (5).

Figures 2 and 3 show the results of the calculation of the self-focusing of a Gaussian pulse

$$\mathscr{E}(z=0,r,t)=\mathscr{E}_0\exp\left(-r^2/a_0^2-t^2/\tau_0^2\right),\quad \rho(z,r,t=0)=\rho_0.$$

Figure 2 shows the axial profiles of the pulse at characteristic times with the following parameters: $\mu = 10$, $\nu = 0.1$, $\varkappa = 1$, $\sigma = 10$, $\mathscr{E}_0 = 0.8$, $\rho_0 = 4.0$, $\tau_0 = 1.6$, and $a_0 = 2$. This value of μ corresponds to the temperature $T_e \sim 1$ eV, which corresponds to a large thermal pressure, i.e., $c > 0$.

As the results indicate, the front part of the pulse self-focuses most effectively. Starting at some time, the effectiveness of the self-focusing decreases. In this case, a

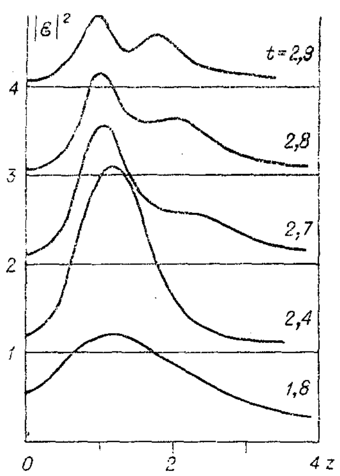

Fig. 3

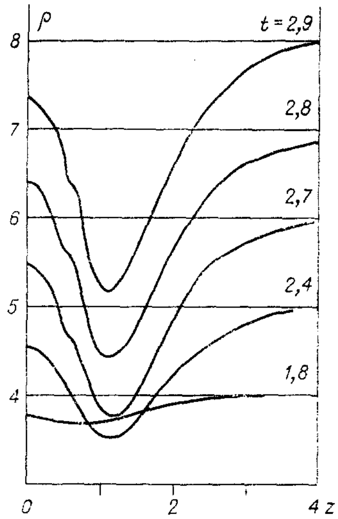

Fig. 4

certain part of the pulse focuses, while the tail end of the pulse may not focus at all. This is seen in Fig. 2, wherein the time t = 2.2 corresponds to the passage of the pulse maximum through the plasma boundary.

As follows from (4), the duration of that part of the pulse falling into the self-focusing regime is of the order of $1/\sqrt{\alpha c}$.

Fig. 3 shows the axial profiles for $\mu = 2$, $\nu = 0.1$, $\varkappa = 1$, $\sigma = 10$, $\mathscr{E}_0 = 0.8$, $\tau_0 = 1.2$, and $a_0 = 2$, which corresponds to the case $c < 0$. In this case, the profile has an oscillatory structure.

It should be noted that in this case self-modulation of the pulse is much weaker in comparison with Kerr and thermal self-focusing. For all of the different variations of the calculations performed, the numer of oscillations did not exceed 2–3. In spite of the fact that for $c < 0$, the depth of the oscillations in the field amplitude was large, the oscillations of the density were either very weak or absent entirely. This is seen in Fig. 4, wherein the density distribution along the pulse axis is presented for the same parameters and at the same times as in Fig. 3.

Figure 5 shows the typical picture of the evolution of the radial distribution in

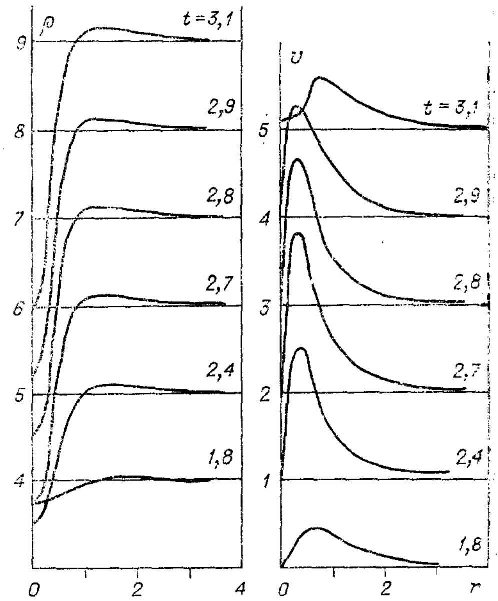

Fig. 5

density and velocity of the plasma for z = 1.1. The starting stages correspond to the appearance of self-focusing and the formation of a waveguide channel under the action of the field. The final stages of this picture, t = 2.9 and t = 3.1, correspond to the free motion of the plasma and collapse of the waveguide channel. The parameters in Fig. 5 are the same as in Figs. 3 and 4.

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SINGULAR SELF-SIMILAR SUPERDENSE COMPRESSION REGIMES FOR LASER TARGETS

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The approach to laser-driven fusion reactions proposed in [1, 2] is based on a special mode for depositing energy in the laser target that ensures compression of matter to densities of the order of 10^3–10^4 times the initial solid density. The optimum choice of laser pulse shape and target parameters on the basis of numerical calculations presents great difficulties. The key idea in the calculations is usually the requirement of adiabatic compression of the dense core of the target. Dimensional analysis then permits establishing the asymptotic law for the increase with time of the mechanical power expended on compression [3]: $E_m \sim |t|^{-2}$ (here and below, we consider spherical compression of matter with an adiabatic index $\gamma = 5/3$; time is measured from the instant of collapse). A particular self-similar solution, satisfying this law, is indicated in [4, 5]. In this case, the following questions remain unclear: 1) Does the self-similar correspond to the only optimum compression regime and are flows close to self-similar flows realized with the numerical simulation? 2) How is the laser pulse shape related to the time dependence of the mechanical power? In the present work, it is shown that the solution in [4, 5] is not the only solution in the sense indicated and two new families of self-similar solutions are constructed to the equations of gasdynamics, describing the compression of simple shell-like and continuous uniform laser targets. The solutions constructed are singular; the corresponding values of the self-similar indicators lie within some interval of acceptable values. In order to construct the solutions, it is necessary to transform to a scale-invariant representation of the hydrodynamic variables. The reverse procedure for calculating the physical quantities requires the characteristic parameters of the medium: the specific entropy in the case of shells and the initial plasma density in the case of continuous targets. The solutions constructed describe the process of an unbounded concentration of energy as the instant of collapse is approached; in an actual experiment, the magnitude of the total energy, of course, is limited and determines the maximum degree of compression. It is shown by way of comparison with numerical calculations that for a correct choice of parameters the self-similar solutions found give a quantitative description of the dynamics of the compression of the dense core of a target in regimes that are similar to those studied numerically [1, 2]. It has been found that for shells with degrees of compression of practical interest, the law that describes the change in power can differ noticeably from the asymptotic law.

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